Dec 10, 2024
Period 4

$$\sin\frac{\pi}{3} = \sin 60° = y = \frac{y}{1}$$

$$\cos\frac{\pi}{3} = \cos 60° = x = \frac{x}{1}.$$

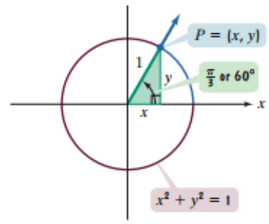


Figure 4.31(b) (repeated)

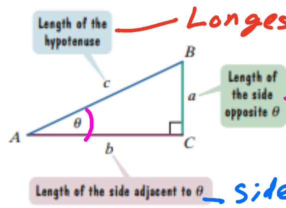


Figure 4.32

Longest Side

Side That does not Touch angle

Side That does Touch The angle, but is not The Hypotenuse

Right Triangle Definitions of Trigonometric Functions

See **Figure 4.32**. The six **trigonometric functions of the acute angle θ** are defined as follows:

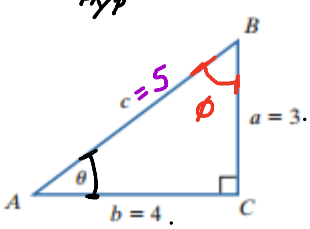
$$\sin\theta = \frac{\text{length of side opposite angle }\theta}{\text{length of hypotenuse}} = \frac{a}{c} \qquad \csc\theta = \frac{\text{length of hypotenuse}}{\text{length of side opposite angle }\theta} = \frac{c}{a}$$

$$\cos\theta = \frac{\text{length of side adjacent to angle }\theta}{\text{length of hypotenuse}} = \frac{b}{c} \qquad \sec\theta = \frac{\text{length of hypotenuse}}{\text{length of side adjacent to angle }\theta} = \frac{c}{b}$$

$$\tan\theta = \frac{\text{length of side opposite angle }\theta}{\text{length of side adjacent to angle }\theta} = \frac{a}{b} \qquad \cot\theta = \frac{\text{length of side adjacent to angle }\theta}{\text{length of side opposite angle }\theta} = \frac{b}{a}.$$

$\sin\theta = \frac{opp}{Hyp}$

$\sec\theta = \frac{1}{\cos\theta}$



$c^2 = 4^2 + 3^2 = 16 + 9$

$c^2 = 25$

$c = 5$

$\sin\theta = \frac{a}{c} = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

$\sec\theta = \frac{5}{4}$

$\csc\theta = \frac{5}{3}$

$\cot\theta = \frac{4}{3}$

$\cot\theta = \frac{1}{\tan\theta}$

$\sin\phi = \frac{4}{5}$ $\quad \csc\phi = \frac{5}{4}$

$\cos\phi = \frac{3}{5}$ $\quad \sec\phi = \frac{5}{3}$

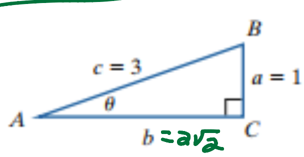
$\tan\phi = \frac{4}{3}$ $\quad \cot\phi = \frac{3}{4}$

$\theta + \phi = 90° = \frac{\pi}{2}$

$\sin\theta = \cos\phi = \frac{3}{5}$

$\cos\theta = \sin\phi = \frac{4}{5}$

$\tan\theta = \cot\phi = \frac{3}{4}$



$1^2 + b^2 = 3^2$

$1 + b^2 = 9$

$b^2 = 8$

$b = \sqrt{8} = \sqrt{2\cdot2\cdot2} = 2\sqrt{2}$

$\sin\theta = \frac{1}{3}$

$\cos\theta = \frac{2\sqrt{2}}{3}$

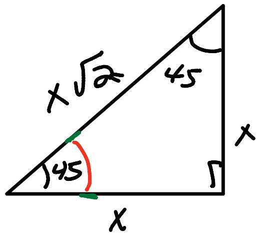
$\tan\theta = \frac{1\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{2}}{4}$

$\sec\theta = \frac{3\cdot\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\csc\theta = 3 = \frac{3}{1}$

$\cot\theta = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$

find sin 45°, cos 45°, and tan 45°.



$\sin 45 = \frac{\cancel{x}}{\cancel{x}\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45 = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45 = \frac{x}{x} = 1$

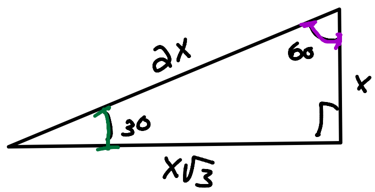
find sin 60°, cos 60°, sin 30°, and cos 30°.

$\sin 60 = \frac{\cancel{x}\sqrt{3}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{\cancel{x}}{2\cancel{x}} = \frac{1}{2}$

$\sin 30° = \frac{\cancel{x}}{2x} = \frac{1}{2}$

$\cos 30° = \frac{\cancel{x}\sqrt{3}}{2\cancel{x}} = \frac{\sqrt{3}}{2}$



$\sin 60 = \cos 30$

$\cos 60 = \sin 30$

$\sin 60 = \cos(90-60) = \cos 30$

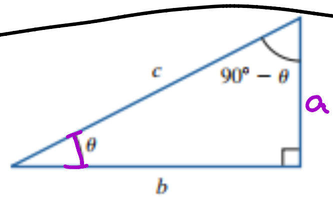


Figure 4.38

$$\sin\theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{a}{c}$$

$$\cos(90° - \theta) = \frac{\text{length of side adjacent to } (90° - \theta)}{\text{length of hypotenuse}} = \frac{a}{c}.$$

Thus, $\sin\theta = \cos(90° - \theta)$.

$\cos\theta = \sin(90-\theta)$

$\tan\theta = \frac{a}{b} = \cot(90-\theta)$

cofunctions.

Any pair of trigonometric functions f and g for which

$$f(\theta) = g(90° - \theta) \quad \text{and} \quad g(\theta) = f(90° - \theta)$$

$\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{\pi}{3} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Find a cofunction with the same value as the given expression:

a. sin 72° **b.** csc $\frac{\pi}{3}$.

$\sin 72° = 0.95$

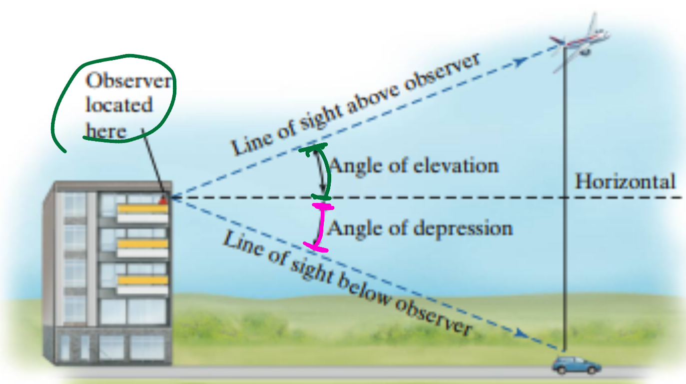
$\cos(90-72) = \cos 18° = 0.95$

$\csc\frac{\pi}{3} = \frac{1}{\sin\frac{\pi}{3}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\left(\frac{\pi}{2}-\frac{\pi}{3}\right) = \sec\frac{\pi}{6} = \frac{1}{\cos\frac{\pi}{6}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

$\frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6}$

$\sec\frac{\pi}{6} = \csc\frac{\pi}{3}$



Sighting the top of a building, a surveyor measured the angle of elevation to be 22°. The transit is 5 feet above the ground and 300 feet from the building. Find the building's height.

To angle with 22°

300 FT = adj side

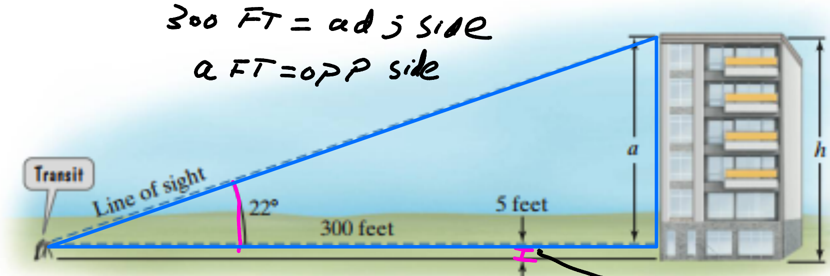
a FT = opp side

$\tan 22° = \frac{a}{300}$

$300 \cdot 0.40402623 = \frac{a}{300} \cdot 300$

121.2 FT = a

height = 121.2 + 5 = 126.2



CHECK POINT 6 The irregular blue shape in **Figure 4.41** represents a lake. The distance across the lake, a, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?

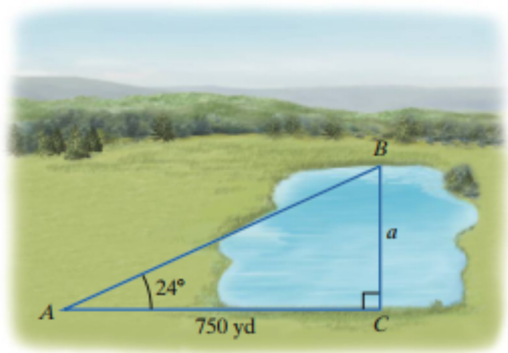


Figure 4.41

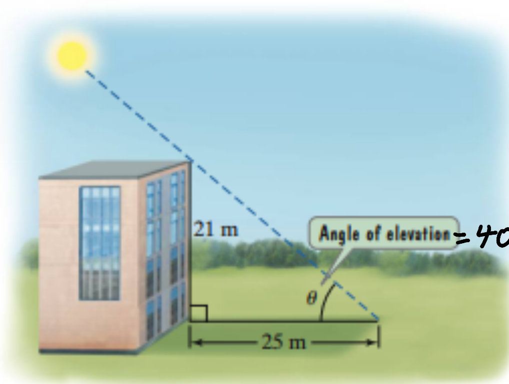
$$\tan 24 = \frac{a}{750\text{yd}}$$

$$750 \cdot 0.4452286853 = \frac{a}{750} \cdot 750$$

$$333.9 = a$$

A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the Sun to the nearest degree.

Solution The situation is illustrated in **Figure 4.42**. We are asked to find θ.



= 40.0303°

$$\tan\theta = \frac{21}{25} = .84$$

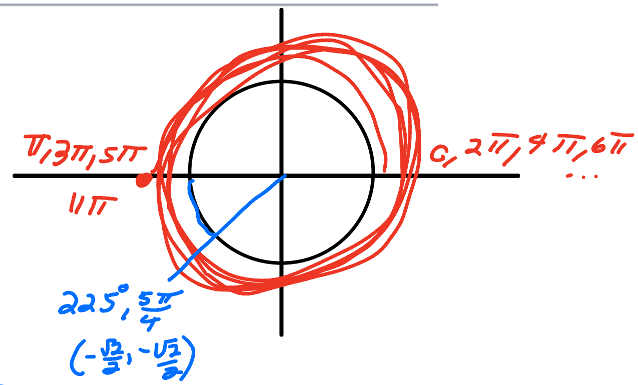
$$\tan^{-1}\frac{21}{25} = 40.0303^\circ$$

$$\tan 40.0303 = 0.84$$

Figure 4.42

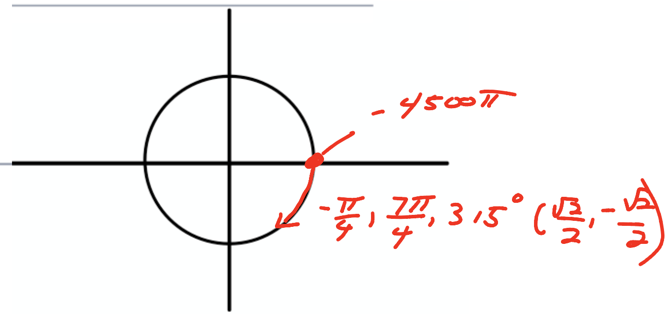
Find the exact value of the given trigonometric function. Do not use a calculator.

$$-\cot\left(\frac{\pi}{4}+11\pi\right) = -\frac{\cos\left(\frac{\pi}{4}+11\pi\right)}{\sin\left(\frac{\pi}{4}+11\pi\right)} = -\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$$



Find the exact value of the given trigonometric function. Do not use a calculator.

$$\cos\left(-\frac{\pi}{4}-4500\pi\right) = \frac{\sqrt{2}}{2}$$



Let **sin** t = a, **cos** t = b, and **tan** t = c. Write the expression in terms of a, b, and c.

$(T-3200\pi)$

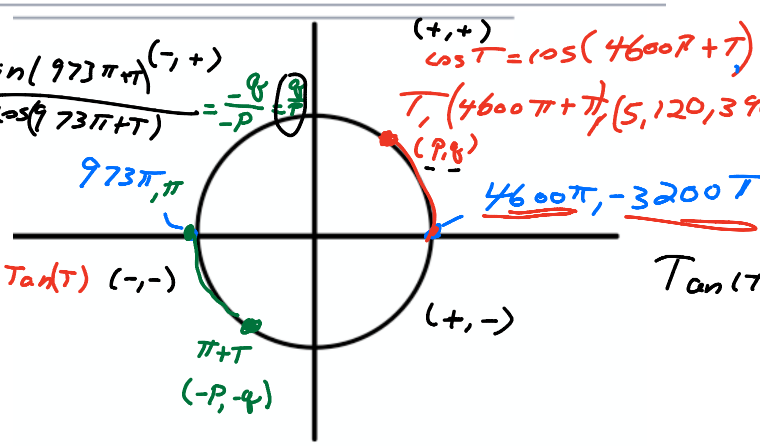
cos t + **cos** (t + 4600π) − **tan** t − **tan** (t + 973π) − **sin** t + 3 **sin** (t − 3200π)

$\cos T + \cos T - \tan T - \tan T - \sin T + 3\sin T = b + b - c - c - a + 3a = 2b - 2c + 2a$

$$\tan(\pi + 973\pi) = \frac{\sin(973\pi+\pi)}{\cos(973\pi+T)} \; (-,+) = \frac{-q}{-p} = \frac{q}{p}$$

$(+,+)$ $\cos T = \cos(4600\pi + T)$

$T, (4600\pi + T), (5{,}120{,}396\pi + T)$

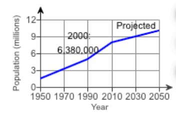


$$\tan(973\pi + T) = \frac{q}{p} = \tan(T)$$

$$\tan(T) = \frac{q}{p}$$

a. In 2000, the population of a country was approximately 6.38 million and by 2060 it is projected to grow to 11 million. Use the exponential growth model $A = A_0e^{kt}$, in which t is the number of years after 2000 and A_0 is in millions, to find an exponential growth function that models the data.

b. By which year will the population be 12 million?



a. The exponential growth function that models the data is $A = 6.38e^{0.01t}$.
(Simplify your answer. Use integers or decimals for any numbers in the expression. Round to two decimal places as needed.)

b. The country's population will be 12 million in the year ☐.
(Round to the nearest year as needed.)

T	P
2000=0	6.38
2060=60	11

$6.38 = A_0e^{k\cdot 0} = A_0\cdot e^0 = A_0\cdot 1$

$A_0 = 6.38$

$P = 6.38e^{kT}$

$\dfrac{11}{6.38} = \dfrac{6.38e^{k\cdot 60}}{6.38} \Rightarrow 1.72 = e^{60k}$

$\ln 1.72 = \ln e^{60k}$

$0.544727 = 60k\cdot \ln e$

$\dfrac{0.544727}{60} = \dfrac{60k\cdot 1}{60}$

$0.00908 = k$

$A = 6.38e^{0.00908T}$

$12 = 6.38e^{0.00908T}$

$1.88088 = e^{0.00908T}$

$\ln 1.88089 = \ln e^{0.00908T}$

$\dfrac{0.63173855}{0.00908} = \dfrac{0.00908T\underbrace{\ln e}_{1}}{0.00908}$

64.57 Years

Determine whether the following statement makes sense or does not make sense, and explain your reasoning.

Because **sec** $\frac{\pi}{4} = \sqrt{2}$, I can conclude that **sec** $\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Select the correct choice below and fill in the answer box to complete your choice.

$\cos T$, $\sec T$ } Even Fun

$\cos(-T) = \cos T$

$\sec(-T) = \sec T$

- A. The statement makes sense because **sec** $(-t)=$ ☐.
- B. The statement does not make sense because **sec** $(-t)=$ **sec** t.

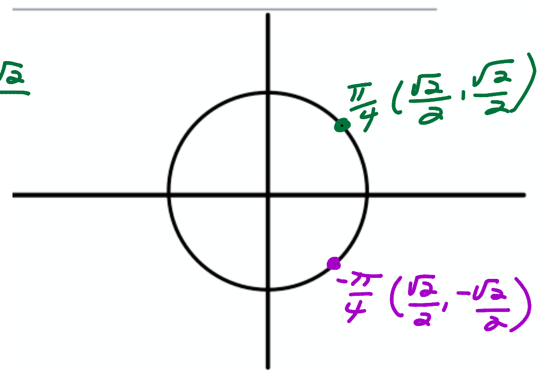
$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\sec\frac{\pi}{4} = \frac{2\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

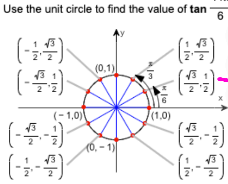
$\sec\frac{\pi}{4} = \sqrt{2}$

$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sec -\frac{\pi}{4} = \sqrt{2}$



Use the unit circle to find the value of **tan** $\frac{11\pi}{6}$ and even or odd trigonometric functions to find the value of **tan** $\left(-\frac{11\pi}{6}\right)$.



$\frac{\pi}{6}, -\frac{11\pi}{6}$

$\leftarrow \frac{11\pi}{6}$

$\tan\frac{11\pi}{6} = \frac{\sin\frac{11\pi}{6}}{\cos\frac{11\pi}{6}} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}} = \frac{-1\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{-\sqrt{3}}{3}$

$\tan\left(\frac{-11\pi}{6}\right) = \frac{\sin\frac{-11\pi}{6}}{\cos\frac{-11\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$

Select the correct choice below and fill in any answer boxes within your choice.

- A. $\tan\frac{11\pi}{6} = \boxed{\frac{-\sqrt{3}}{3}}$

  (Type an exact answer, using radicals as needed. Simplify your answer. Rationalize the denominator.)
- B. The expression is undefined.

Select the correct choice below and fill in any answer boxes within your choice.

- A. $\tan\left(-\frac{11\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{3}}$

  (Type an exact answer, using radicals as needed. Simplify your answer. Rationalize the denominator.)
- B. The expression is undefined.